QUESTION 2011

Group – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following:

i) The degree of the polynomial $(x^2+x-2)/(x-1)$ is

a) 0 ✓b) 1 c) 2 d) 3

ii) If G be a group and $a, b \in G$. Then $(a^{-1}b)^{-1}$ is equal to

a) ab^{-1} ✓b) $b^{-1}a$ c) $a^{-1}b^{-1}$ d) $b^{-1}a^{-1}$

iii) $\frac{\partial}{\partial x}(x^y) =$

a) 1 b) yx^y c) $x^y \log x$ ✓d) yx^{y-1}

iv) If $P=\{2,4,6,7,8,9\}$, $Q=\{1,2,6,9\}$ then $P \cap Q$ is

a) $\{1,2,6\}$ ✓b) $\{2,6,9\}$ c) $\{1,6,9\}$ d) $\{4,6,9\}$

v) The value of $\lim_{x\to 3}\frac{x^3-3^3}{x-3}$ is

a) – 12 b) 12 ✓c) 27 d) – 27

vi) If A be a matrix whose inverse exists then which of the following is not true?

a) $(A^T)^{-1} = (A^{-1})^T$ ✓b) $A^{-1} = (\det(A))^{-1}$

c) $(A^2)^{-1} = (A^{-1})^2$ d) none of these

vii) The equation $x^4+2x^2-7x-5=0$ has

a) one real roots and three complex roots
b) one complex roots and three real roots
✓c) two real roots and two complex roots
d) four real roots

viii) Cardan's method is used for solving equation of degree

a) 2
✓b) 3
c) 4
d) none of these

ix) If α, β, γ be the roots of $x^3-3x^2+6x-2=0$, then $\sum\alpha\beta$ is

a) 3
✓b) 6
c) 2
d) none of these

x) $f(x, y)=\sqrt{x}+\sqrt{y}$ is a function of degree

✓a) $\frac{1}{2}$
b) $\frac{1}{3}$
c) 0
d) $\frac{1}{4}$

xi) The equation $r=3\sin\theta+4\cos\theta$ represents

a) a parabola
b) an ellipse
c) a straight line
✓d) a circle

xii) The inverse of the matrix $\begin{bmatrix}2 & 3\\ 4 & 6\end{bmatrix}$ is

a) $\begin{bmatrix}2 & -3\\ 4 & 6\end{bmatrix}$
b) $\begin{bmatrix}1 & 2\\ -\frac{3}{2} & 3\end{bmatrix}$
c) $\begin{bmatrix}-2 & 4\\ -3 & 6\end{bmatrix}$
✓d) does not exist

Group – B
(Short Answer Type Questions)

2. Prove that the set of real numbers of the form $a+b\sqrt{2}$ where a and b are rational numbers, forms a field under addition and multiplication.

See Topic: BINARY COMPOSITION, Short Answer Type Question No. 5.

3. Solve the equation $x^3-9x^2+14x+24=0$, two of whose roots are in the ratio $3:2$.

See Topic: POLYNOMIAL, Short Answer Type Question No. 16.

4. Prove that, any square matrix can be expressed assume of a symmetric matrix and a skew symmetric matrix.

See Topic: MATRICES, Short Answer Type Question No. 10.

5. If $u = \tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\frac{1}{4}\sin 2u$.

See Topic: FUNCTION OF SEVERAL VARIABLES, Long Answer Type Question No. 3.

6. A function $f(x)$ is defined as follows

$f(x)=1+x$ when $x\le 2$.

$=5-x$ when $x>2$

Show that $f(x)$ is continuous at $x=2$ but $f(2)$ does not exist.

See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 10.

Group – C
(Long Answer Type Questions)

7. a) State Descart's rule of sign. Using this rule find the nature of the roots of the equation

$x^4-7x^3+21x^2-9x+21=0$.

b) Solve the following system of linear equations by Cramer's rule

$x-y+2z=1$

$x+y+z=2$

$2x-y+z=5$.

c) If by a transformation of one rectangular axis to another with same origin the expression $ax+by$ changes to $a'x'+b'y'$. Prove that $a^2+b^2=a'^2+b'^2$.

a) See Topic: POLYNOMIAL, Long Answer Type Question No. 10.
b) See Topic: MATRICES, Long Answer Type Question No. 12.
c) See Topic: TRANSFORMATION OF CO-ORDINATES, Long Answer Type Question No. 6.

8. a) Show that the equation $20x^2+15xy+9x+3y+1=0$ represents a pair of intersecting straight lines which are equidistant from the origin.

b) Show that $\cos x>1-\frac{x^2}{2}$ if $0<x<\frac{\pi}{2}$.

c) If α, β, γ be the roots of the equation $x^3-px^2+qx-r=0$, then find the value of $\sum\frac{1}{\alpha}$.

a) See Topic: GENERAL EQUATION OF SECOND DEGREE, Long Answer Type Question No. 6.
b) See Topic: MISCELLANEOUS, Short Answer Type Question No. 5.
c) See Topic: POLYNOMIAL, Short Answer Type Question No. 17.

9. a) If $A=\{a,b,c,d,e\}$, $B=\{c,a,e,g\}$ and $C=\{b,e,f,g\}$, then show that

$$(A\cup B)\cap C=(A\cap C)\cup(B\cap C).$$

b) Reduce the following equation to the canonical form and determine the nature of the conic represented by it $x^2-4xy+4y^2-12x-6y-39=0$

c) Evaluate $\lim\limits_{x\to 1}\left(\dfrac{x}{x-1}-\dfrac{1}{\log x}\right)$.

a) See Topic: SET THEORY, Short Answer Type Question No. 5.

b) See Topic: GENERAL EQUATION OF SECOND DEGREE, Short Answer Type Question No. 1.

c) See Topic: LIMIT, CONTINUITY & DIFFERENTIABILITY, Short Answer Type Question No. 11.

10. a) Evaluate $\int\dfrac{dx}{(1+x)\sqrt{1-x^2}}$.

b) If PSQ be a focal chord of a conic with focus S and semi-latus rectum l, then prove that $\dfrac{1}{SP}+\dfrac{1}{SQ}=\dfrac{2}{l}$.

c) If $A-2B=\begin{bmatrix}0&6&26\\6&-9&12\\2&9&-10\end{bmatrix}$ and $2A+B=\begin{bmatrix}10&-3&4\\12&-3&4\\4&3&0\end{bmatrix}$, find A and B.

a) See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 9.

b) See Topic: POLAR EQUATIONS, Long Answer Type Question No. 1(a).

c) See Topic: MATRICES, Long Answer Type Question No. 1.

11. a) If G be a group such that $(ab)^2=a^2b^2\ \forall a,b\in G$, show that the group G is abelian.

b) Show that $\int_0^1\dfrac{\log(1+x)}{1+x^2}dx=\dfrac{\pi}{8}\log 2$.

c) If $y=e^{-x}\sin x$, then show that $y_4+4y=0$.

a) See Topic: BINARY COMPOSITION, Short Answer Type Question No. 6.

b) See Topic: DEFINITE INTEGRALS, Short Answer Type Question No. 10.

c) See Topic: SUCCESSIVE DIFFERENTIATION, Short Answer Type Question No. 9.